INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY

TOYOTA MOTOR CREDIT CORPORATION (TMCC), a subsidiary of Toyota Motor, offered some securities for sale to the public on March 28, 2008. Under the terms of the deal, TMCC promised to repay the owner of one of these securities $100,000 on March 28, 2038, but investors would receive nothing until then. Investors paid TMCC $24,099 for each of these securities; so, they gave up $24,099 on March 28, 2008, for the promise of a $100,000 payment 30 years later. Such a security, for which you pay some amount today in exchange for a promised lump sum to be received at a future date, is about the simplest possible type. Is giving up $24,099 in exchange for $100,000 in 30 years a good deal? On the plus side, you get back about $4 for every $1 you put up. That probably sounds good; but on the down side, you have to wait 30 years to get it. What you need to know is how to analyze this trade-off; this chapter gives you the tools you need.

LEARNING OBJECTIVES

After studying this chapter, you should understand:

LO1 How to determine the future value of an investment made today.

LO2 How to determine the present value of cash to be received at a future date.

LO3 How to find the return on an investment.

LO4 How long it takes for an investment to reach a desired value.

One of the basic problems faced by the financial manager is how to determine the value today of cash flows expected in the future. For example, the jackpot in a PowerBall™ lottery drawing was $110 million. Does this mean the winning ticket was worth $110 million? The answer is no because the jackpot was actually going to pay out over a 20-year period at a rate of $5.5 million per year. How much was the ticket worth then? The answer depends on the time value of money, the subject of this chapter.

In the most general sense, the phrase *time value of money* refers to the fact that a dollar in hand today is worth more than a dollar promised at some time in the future. On a practical level, one reason for this is that you could earn interest while you waited; so a dollar today would grow to more than a dollar later. The trade-off between money now and money later thus depends on, among other things, the rate you can earn by investing. Our goal in this chapter is to explicitly evaluate this trade-off between dollars today and dollars at some future time.

A thorough understanding of the material in this chapter is critical to understanding material in subsequent chapters, so you should study it with particular care. We will present a number of examples in this chapter. In many problems, your answer may differ from ours slightly. This can happen because of rounding and is not a cause for concern.

Future Value and Compounding

5.1

future value (FV)
The amount an investment is worth after one or more periods.

The first thing we will study is future value. **Future value (FV)** refers to the amount of money an investment will grow to over some period of time at some given interest rate. Put another way, future value is the cash value of an investment at some time in the future. We start out by considering the simplest case: a single-period investment.

INVESTING FOR A SINGLE PERIOD

Suppose you invest \$100 in a savings account that pays 10 percent interest per year. How much will you have in one year? You will have \$110. This \$110 is equal to your original *principal* of \$100 plus \$10 in interest that you earn. We say that \$110 is the future value of \$100 invested for one year at 10 percent, and we simply mean that \$100 today is worth \$110 in one year, given that 10 percent is the interest rate.

In general, if you invest for one period at an interest rate of r, your investment will grow to $(1 + r)$ per dollar invested. In our example, r is 10 percent, so your investment grows to $1 + .10 = 1.1$ dollars per dollar invested. You invested \$100 in this case, so you ended up with $\$100 \times 1.10 = \$110$.

compounding
The process of accumulating interest on an investment over time to earn more interest.

interest on interest
Interest earned on the reinvestment of previous interest payments.

compound interest
Interest earned on both the initial principal and the interest reinvested from prior periods.

simple interest
Interest earned only on the original principal amount invested.

INVESTING FOR MORE THAN ONE PERIOD

Going back to our \$100 investment, what will you have after two years, assuming the interest rate doesn't change? If you leave the entire \$110 in the bank, you will earn $\$110 \times .10 = \$11$ in interest during the second year, so you will have a total of $\$110 + 11 = \$121$. This \$121 is the future value of \$100 in two years at 10 percent. Another way of looking at it is that one year from now you are effectively investing \$110 at 10 percent for a year. This is a single-period problem, so you'll end up with \$1.10 for every dollar invested, or $\$110 \times 1.1 = \$121$ total.

This \$121 has four parts. The first part is the \$100 original principal. The second part is the \$10 in interest you earned in the first year, and the third part is another \$10 you earn in the second year, for a total of \$120. The last \$1 you end up with (the fourth part) is interest you earn in the second year on the interest paid in the first year: $\$10 \times .10 = \$1$.

This process of leaving your money and any accumulated interest in an investment for more than one period, thereby *reinvesting* the interest, is called **compounding**. Compounding the interest means earning **interest on interest**, so we call the result **compound interest**. With **simple interest**, the interest is not reinvested, so interest is earned each period only on the original principal.

Interest on Interest — EXAMPLE 5.1

Suppose you locate a two-year investment that pays 14 percent per year. If you invest \$325, how much will you have at the end of the two years? How much of this is simple interest? How much is compound interest?

At the end of the first year, you will have $\$325 \times (1 + .14) = \$370.50$. If you reinvest this entire amount and thereby compound the interest, you will have $\$370.50 \times 1.14 = \$422.37$ at the end of the second year. The total interest you earn is thus $\$422.37 - 325 = \$97.37$. Your \$325 original principal earns $\$325 \times .14 = \$45.50$ in interest each year, for a two-year total of \$91 in simple interest. The remaining $\$97.37 - 91 = \$6.37$ results from compounding. You can check this by noting that the interest earned in the first year is \$45.50. The interest on interest earned in the second year thus amounts to $\$45.50 \times .14 = \$6.37$, as we calculated.

We now take a closer look at how we calculated the \$121 future value. We multiplied \$110 by 1.1 to get \$121. The \$110, however, was \$100 also multiplied by 1.1. In other words:

$$\begin{aligned}\$121 &= \$110 \times 1.1 \\ &= (\$100 \times 1.1) \times 1.1 \\ &= \$100 \times (1.1 \times 1.1) \\ &= \$100 \times 1.1^2 \\ &= \$100 \times 1.21\end{aligned}$$

At the risk of belaboring the obvious, let's ask: How much would our \$100 grow to after three years? Once again, in two years, we'll be investing \$121 for one period at 10 percent. We'll end up with \$1.10 for every dollar we invest, or $\$121 \times 1.1 = \$133.10$ total. This \$133.10 is thus:

$$\begin{aligned}\$133.10 &= \$121 \times 1.1 \\ &= (\$110 \times 1.1) \times 1.1 \\ &= (\$100 \times 1.1) \times 1.1 \times 1.1 \\ &= \$100 \times (1.1 \times 1.1 \times 1.1) \\ &= \$100 \times 1.1^3 \\ &= \$100 \times 1.331\end{aligned}$$

For a discussion of time value concepts (and lots more) see www.financeprofessor.com.

You're probably noticing a pattern to these calculations, so we can now go ahead and state the general result. As our examples suggest, the future value of \$1 invested for t periods at a rate of r per period is this:

$$\text{Future value} = \$1 \times (1 + r)^t \qquad \textbf{[5.1]}$$

The expression $(1 + r)^t$ is sometimes called the *future value interest factor* (or just *future value factor*) for \$1 invested at r percent for t periods and can be abbreviated as FVIF(r, t).

In our example, what would your \$100 be worth after five years? We can first compute the relevant future value factor as follows:

$$(1 + r)^t = (1 + .10)^5 = 1.1^5 = 1.6105$$

Your \$100 will thus grow to:

$$\$100 \times 1.6105 = \$161.05$$

The growth of your \$100 each year is illustrated in Table 5.1. As shown, the interest earned in each year is equal to the beginning amount multiplied by the interest rate of 10 percent.

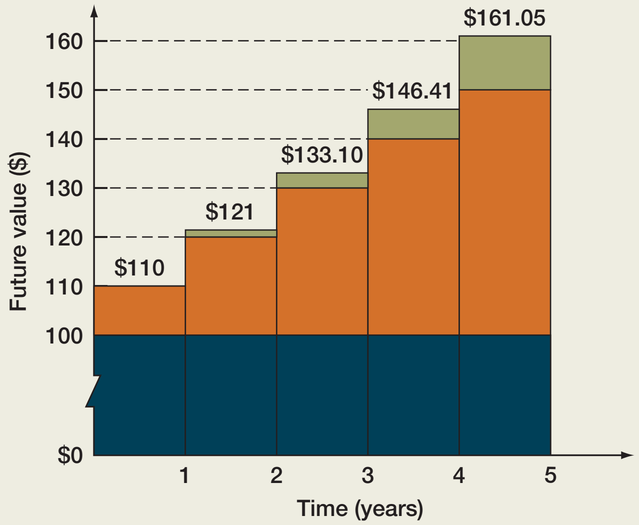
TABLE 5.1

Future Value of \$100 at 10 percent

Year	Beginning Amount	Simple Interest	Compound Interest	Total Interest Earned	Ending Amount
1	\$100.00	\$10	\$ .00	\$10.00	\$110.00
2	110.00	10	1.00	11.00	121.00
3	121.00	10	2.10	12.10	133.10
4	133.10	10	3.31	13.31	146.41
5	146.41	10	4.64	14.64	161.05
		Total \$50 simple interest	Total \$11.05 compound interest	Total \$61.05 interest	

FIGURE 5.1

Future Value, Simple Interest, and Compound Interest



Growth of $100 original amount at 10 percent per year. The orange shaded area shows the simple interest. The green shaded area represents the portion of the total that results from compounding of interest.

In Table 5.1, notice the total interest you earn is $61.05. Over the five-year span of this investment, the simple interest is $100 × .10 = $10 per year, so you accumulate $50 this way. The other $11.05 is from compounding.

A brief introduction to key financial concepts is available at www.teachmefinance.com.

Figure 5.1 illustrates the growth of the compound interest in Table 5.1. Notice how the simple interest is constant each year, but the amount of compound interest you earn gets bigger every year. The amount of the compound interest keeps increasing because more and more interest builds up and there is thus more to compound.

Future values depend critically on the assumed interest rate, particularly for long-lived investments. Figure 5.2 illustrates this relationship by plotting the growth of $1 for different rates and lengths of time. Notice the future value of $1 after 10 years is about $6.20 at a 20 percent rate, but it is only about $2.60 at 10 percent. In this case, doubling the interest rate more than doubles the future value.

To solve future value problems, we need to come up with the relevant future value factors. There are several different ways of doing this. In our example, we could have multiplied 1.1 by itself five times. This would work just fine, but it would get to be very tedious for, say, a 30-year investment.

Fortunately, there are several easier ways to get future value factors. Most calculators have a key labeled "y^x". You can usually just enter 1.1, press this key, enter 5, and press the "=" key to get the answer. This is an easy way to calculate future value factors because it's quick and accurate.

Alternatively, you can use a table that contains future value factors for some common interest rates and time periods. Table 5.2 contains some of these factors. Table A.1 in the appendix at the end of the book contains a much larger set. To use the table, find the column that corresponds to 10 percent. Then look down the rows until you come to five periods. You should find the factor that we calculated, 1.6105.

Tables such as 5.2 are not as common as they once were because they predate inexpensive calculators and are available only for a relatively small number of rates. Interest rates

FIGURE 5.2

Future Value of $1 for Different Periods and Rates

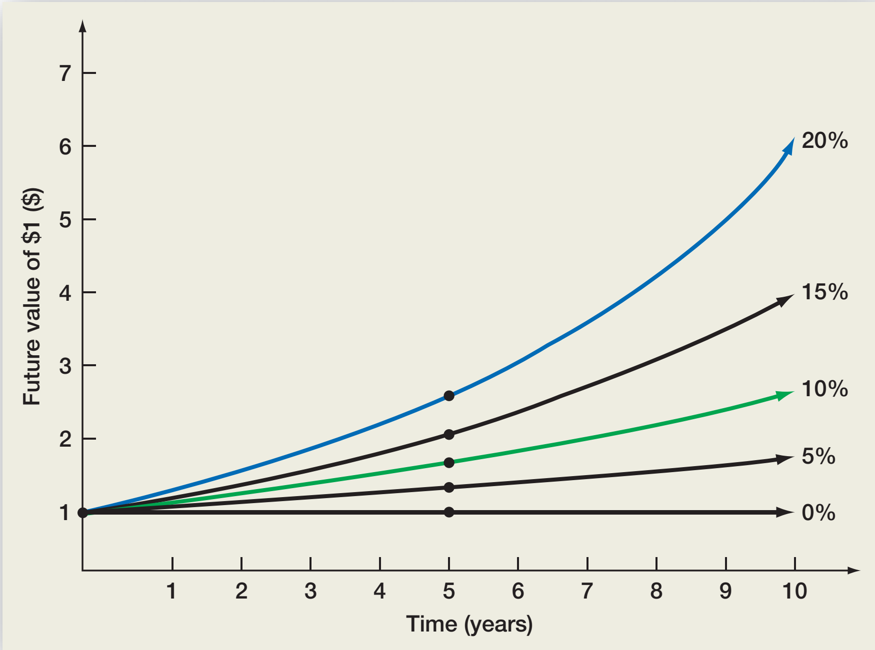


TABLE 5.2

Future Value Interest Factors

	Interest Rate			
Number of Periods	5%	10%	15%	20%
1	1.0500	1.1000	1.1500	1.2000
2	1.1025	1.2100	1.3225	1.4400
3	1.1576	1.3310	1.5209	1.7280
4	1.2155	1.4641	1.7490	2.0736
5	1.2763	1.6105	2.0114	2.4883

are often quoted to three or four decimal places, so the tables needed to deal with these accurately would be quite large. As a result, the real world has moved away from using them. We will emphasize the use of a calculator in this chapter.

These tables still serve a useful purpose. To make sure you are doing the calculations correctly, pick a factor from the table and then calculate it yourself to see that you get the same answer. There are plenty of numbers to choose from.

EXAMPLE 5.2 Compound Interest

You've located an investment that pays 12 percent per year. That rate sounds good to you, so you invest $400. How much will you have in three years? How much will you have in seven years? At the end of seven years, how much interest will you have earned? How much of that interest results from compounding?

Based on our discussion, we can calculate the future value factor for 12 percent and three years as follows:

$$(1 + r)^t = 1.12^3 = 1.4049$$

Your \$400 thus grows to:

$$\$400 \times 1.4049 = \$561.97$$

After seven years, you will have:

$$\$400 \times 1.12^7 = \$400 \times 2.2107 = \$884.27$$

Thus, you will more than double your money over seven years.

Because you invested \$400, the interest in the \$884.27 future value is \$884.27 − 400 = \$484.27. At 12 percent, your \$400 investment earns \$400 × .12 = \$48 in simple interest every year. Over seven years, the simple interest thus totals 7 × \$48 = \$336. The other \$484.27 − 336 = \$148.27 is from compounding.

The effect of compounding is not great over short time periods, but it really starts to add up as the horizon grows. To take an extreme case, suppose one of your more frugal ancestors had invested \$5 for you at a 6 percent interest rate 200 years ago. How much would you have today? The future value factor is a substantial $1.06^{200} = 115{,}125.90$ (you won't find this one in a table), so you would have \$5 × 115,125.90 = \$575,629.52 today. Notice that the simple interest is just \$5 × .06 = \$.30 per year. After 200 years, this amounts to \$60. The rest is from reinvesting. Such is the power of compound interest!

How Much for That Island? — EXAMPLE 5.3

To further illustrate the effect of compounding for long horizons, consider the case of Peter Minuit and the American Indians. In 1626, Minuit bought all of Manhattan Island for about \$24 in goods and trinkets. This sounds cheap, but the Indians may have gotten the better end of the deal. To see why, suppose the Indians had sold the goods and invested the \$24 at 10 percent. How much would it be worth today?

About 385 years have passed since the transaction. At 10 percent, \$24 will grow by quite a bit over that time. How much? The future value factor is roughly:

$$(1 + r)^t = 1.1^{385} \approx 8{,}600{,}000{,}000{,}000{,}000$$

That is, 8.6 followed by 14 zeroes. The future value is thus on the order of \$24 × 8.6 = \$207 *quadrillion* (give or take a few hundreds of trillions).

Well, \$207 quadrillion is a lot of money. How much? If you had it, you could buy the United States. All of it. Cash. With money left over to buy Canada, Mexico, and the rest of the world, for that matter.

This example is something of an exaggeration, of course. In 1626, it would not have been easy to locate an investment that would pay 10 percent every year without fail for the next 385 years.

CALCULATOR HINTS

Using a Financial Calculator

Although there are the various ways of calculating future values we have described so far, many of you will decide that a financial calculator is the way to go. If you are planning on using one, you should read this extended hint; otherwise, skip it.

A financial calculator is simply an ordinary calculator with a few extra features. In particular, it knows some of the most commonly used financial formulas, so it can directly compute things like future values.

Financial calculators have the advantage that they handle a lot of the computation, but that is really all. In other words, you still have to understand the problem; the calculator just does some of the arithmetic. In fact, there

(*continued*)

is an old joke (somewhat modified) that goes like this: Anyone can make a mistake on a time value of money problem, but to really screw one up takes a financial calculator! We therefore have two goals for this section. First, we'll discuss how to compute future values. After that, we'll show you how to avoid the most common mistakes people make when they start using financial calculators.

How to Calculate Future Values with a Financial Calculator

Examining a typical financial calculator, you will find five keys of particular interest. They usually look like this:

N **I/Y** **PMT** **PV** **FV**

For now, we need to focus on four of these. The keys labeled **PV** and **FV** are just what you would guess: present value and future value. The key labeled **N** refers to the number of periods, which is what we have been calling *t*. Finally, **I/Y** stands for the interest rate, which we have called *r*.[1]

If we have the financial calculator set up right (see our next section), then calculating a future value is very simple. Take a look back at our question involving the future value of $100 at 10 percent for five years. We have seen that the answer is $161.05. The exact keystrokes will differ depending on what type of calculator you use, but here is basically all you do:

1. Enter —100. Press the **PV** key. (The negative sign is explained in the next section.)
2. Enter 10. Press the **I/Y** key. (Notice that we entered 10, not .10; see the next section.)
3. Enter 5. Press the **N** key.

Now we have entered all of the relevant information. To solve for the future value, we need to ask the calculator what the FV is. Depending on your calculator, either you press the button labeled "CPT" (for compute) and then press **FV**, or you just press **FV**. Either way, you should get 161.05. If you don't (and you probably won't if this is the first time you have used a financial calculator!), we will offer some help in our next section.

Before we explain the kinds of problems you are likely to run into, we want to establish a standard format for showing you how to use a financial calculator. Using the example we just looked at, in the future, we will illustrate such problems like this:

Enter	5	10		−100	
	N	**I/Y**	**PMT**	**PV**	**FV**
Solve for					161.05

Here is an important tip: Appendix D contains more detailed instructions for the most common types of financial calculators. See if yours is included; if it is, follow the instructions there if you need help. Of course, if all else fails, you can read the manual that came with the calculator.

How to Get the Wrong Answer Using a Financial Calculator

There are a couple of common (and frustrating) problems that cause a lot of trouble with financial calculators. In this section, we provide some important *dos* and *don'ts*. If you just can't seem to get a problem to work out, you should refer back to this section.

There are two categories we examine: three things you need to do only once and three things you need to do every time you work a problem. The things you need to do just once deal with the following calculator settings:

1. *Make sure your calculator is set to display a large number of decimal places*. Most financial calculators display only two decimal places; this causes problems because we frequently work with numbers—like interest rates—that are very small.
2. *Make sure your calculator is set to assume only one payment per period or per year*. Most financial calculators assume monthly payments (12 per year) unless you say otherwise.
3. *Make sure your calculator is in "end" mode*. This is usually the default, but you can accidently change to "begin" mode.

[1] The reason financial calculators use N and I/Y is that the most common use for these calculators is determining loan payments. In this context, *N* is the number of payments and *I/Y* is the interest rate on the loan. But as we will see, there are many other uses of financial calculators that don't involve loan payments and interest rates.

If you don't know how to set these three things, see Appendix D or your calculator's operating manual. There are also three things you need to do *every time you work a problem*:

1. *Before you start, completely clear out the calculator*. This is very important. Failure to do this is the number one reason for wrong answers; you must get in the habit of clearing the calculator every time you start a problem. How you do this depends on the calculator (see Appendix D), but you must do more than just clear the display. For example, on a Texas Instruments BA II Plus you must press **2nd** then **CLR TVM** for *clear time value of money*. There is a similar command on your calculator. Learn it!

   Note that turning the calculator off and back on won't do it. Most financial calculators remember everything you enter, even after you turn them off. In other words, they remember all your mistakes unless you explicitly clear them out. Also, if you are in the middle of a problem and make a mistake, *clear it out and start over*. Better to be safe than sorry.
2. *Put a negative sign on cash outflows*. Most financial calculators require you to put a negative sign on cash outflows and a positive sign on cash inflows. As a practical matter, this usually just means that you should enter the present value amount with a negative sign (because normally the present value represents the amount you give up today in exchange for cash inflows later). By the same token, when you solve for a present value, you shouldn't be surprised to see a negative sign.
3. *Enter the rate correctly*. Financial calculators assume that rates are quoted in percent, so if the rate is .08 (or 8 percent), you should enter 8, not .08.

If you follow these guidelines (especially the one about clearing out the calculator), you should have no problem using a financial calculator to work almost all of the problems in this and the next few chapters. We'll provide some additional examples and guidance where appropriate.

A NOTE ABOUT COMPOUND GROWTH

If you are considering depositing money in an interest-bearing account, then the interest rate on that account is just the rate at which your money grows, assuming you don't remove any of it. If that rate is 10 percent, then each year you simply have 10 percent more money than you had the year before. In this case, the interest rate is just an example of a compound growth rate.

The way we calculated future values is actually quite general and lets you answer some other types of questions related to growth. For example, your company currently has 10,000 employees. You've estimated that the number of employees grows by 3 percent per year. How many employees will there be in five years? Here, we start with 10,000 people instead of dollars, and we don't think of the growth rate as an interest rate, but the calculation is exactly the same:

$$10{,}000 \times 1.03^5 = 10{,}000 \times 1.1593 = 11{,}593 \text{ employees}$$

There will be about 1,593 net new hires over the coming five years.

To give another example, according to Value Line (a leading supplier of business information for investors), Walmart's 2010 sales were about \$423 billion. Suppose sales are projected to increase at a rate of 15 percent per year. What will Walmart's sales be in the year 2015 if this is correct? Verify for yourself that the answer is about \$851 billion—just over twice as large.

Dividend Growth — EXAMPLE 5.4

The TICO Corporation currently pays a cash dividend of \$5 per share. You believe the dividend will be increased by 4 percent each year indefinitely. How big will the dividend be in eight years?

Here we have a cash dividend growing because it is being increased by management; but once again the calculation is the same:

$$\text{Future value} = \$5 \times 1.04^8 = \$5 \times 1.3686 = \$6.84$$

The dividend will grow by \$1.84 over that period. Dividend growth is a subject we will return to in a later chapter.

Concept Questions

5.1a What do we mean by the future value of an investment?

5.1b What does it mean to compound interest? How does compound interest differ from simple interest?

5.1c In general, what is the future value of $1 invested at *r* per period for *t* periods?

5.2 Present Value and Discounting

When we discuss future value, we are thinking of questions like: What will my $2,000 investment grow to if it earns a 6.5 percent return every year for the next six years? The answer to this question is what we call the future value of $2,000 invested at 6.5 percent for six years (verify that the answer is about $2,918).

Another type of question that comes up even more often in financial management is obviously related to future value. Suppose you need to have $10,000 in 10 years, and you can earn 6.5 percent on your money. How much do you have to invest today to reach your goal? You can verify that the answer is $5,327.26. How do we know this? Read on.

THE SINGLE-PERIOD CASE

present value (PV)
The current value of future cash flows discounted at the appropriate discount rate.

We've seen that the future value of $1 invested for one year at 10 percent is $1.10. We now ask a slightly different question: How much do we have to invest today at 10 percent to get $1 in one year? In other words, we know the future value here is $1, but what is the **present value (PV)**? The answer isn't too hard to figure out. Whatever we invest today will be 1.1 times bigger at the end of the year. Because we need $1 at the end of the year:

$$\text{Present value} \times 1.1 = \$1$$

Or solving for the present value:

$$\text{Present value} = \$1/1.1 = \$.909$$

In this case, the present value is the answer to the following question: What amount, invested today, will grow to $1 in one year if the interest rate is 10 percent? Present value is thus just the reverse of future value. Instead of compounding the money forward into the future, we **discount** it back to the present.

discount
Calculate the present value of some future amount.

EXAMPLE 5.5 Single-Period PV

Suppose you need $400 to buy textbooks next year. You can earn 7 percent on your money. How much do you have to put up today?

We need to know the PV of $400 in one year at 7 percent. Proceeding as in the previous example:

$$\text{Present value} \times 1.07 = \$400$$

We can now solve for the present value:

$$\text{Present value} = \$400 \times (1/1.07) = \$373.83$$

Thus, $373.83 is the present value. Again, this just means that investing this amount for one year at 7 percent will give you a future value of $400.

From our examples, the present value of \$1 to be received in one period is generally given as follows:

$$\text{PV} = \$1 \times [1/(1 + r)] = \$1/(1 + r)$$

We next examine how to get the present value of an amount to be paid in two or more periods into the future.

PRESENT VALUES FOR MULTIPLE PERIODS

Suppose you need to have \$1,000 in two years. If you can earn 7 percent, how much do you have to invest to make sure you have the \$1,000 when you need it? In other words, what is the present value of \$1,000 in two years if the relevant rate is 7 percent?

Based on your knowledge of future values, you know the amount invested must grow to \$1,000 over the two years. In other words, it must be the case that:

$$\begin{aligned}\$1{,}000 &= \text{PV} \times 1.07 \times 1.07\\ &= \text{PV} \times 1.07^2\\ &= \text{PV} \times 1.1449\end{aligned}$$

Given this, we can solve for the present value:

$$\text{Present value} = \$1{,}000/1.1449 = \$873.44$$

Therefore, \$873.44 is the amount you must invest to achieve your goal.

Saving Up — EXAMPLE 5.6

You would like to buy a new automobile. You have \$50,000 or so, but the car costs \$68,500. If you can earn 9 percent, how much do you have to invest today to buy the car in two years? Do you have enough? Assume the price will stay the same.

What we need to know is the present value of \$68,500 to be paid in two years, assuming a 9 percent rate. Based on our discussion, this is:

$$\text{PV} = \$68{,}500/1.09^2 = \$68{,}500/1.1881 = \$57{,}655.08$$

You're still about \$7,655 short, even if you're willing to wait two years.

As you have probably recognized by now, calculating present values is quite similar to calculating future values, and the general result looks much the same. The present value of \$1 to be received t periods into the future at a discount rate of r is:

$$\text{PV} = \$1 \times [1/(1 + r)^t] = \$1/(1 + r)^t \qquad [5.2]$$

The quantity in brackets, $1/(1 + r)^t$, goes by several different names. Because it's used to discount a future cash flow, it is often called a *discount factor*. With this name, it is not surprising that the rate used in the calculation is often called the **discount rate**. We will tend to call it this in talking about present values. The quantity in brackets is also called the *present value interest factor* (or just *present value factor*) for \$1 at r percent for t periods and is sometimes abbreviated as PVIF(r, t). Finally, calculating the present value of a future cash flow to determine its worth today is commonly called **discounted cash flow (DCF) valuation**.

discount rate
The rate used to calculate the present value of future cash flows.

discounted cash flow (DCF) valuation
Calculating the present value of a future cash flow to determine its value today.

To illustrate, suppose you need \$1,000 in three years. You can earn 15 percent on your money. How much do you have to invest today? To find out, we have to determine the

TABLE 5.3

Present Value Interest Factors

Number of Periods	Interest Rate 5%	10%	15%	20%
1	.9524	.9091	.8696	.8333
2	.9070	.8264	.7561	.6944
3	.8638	.7513	.6575	.5787
4	.8227	.6830	.5718	.4823
5	.7835	.6209	.4972	.4019

present value of \$1,000 in three years at 15 percent. We do this by discounting \$1,000 back three periods at 15 percent. With these numbers, the discount factor is:

$$1/(1 + .15)^3 = 1/1.5209 = .6575$$

The amount you must invest is thus:

$$\$1{,}000 \times .6575 = \$657.50$$

We say that \$657.50 is the present or discounted value of \$1,000 to be received in three years at 15 percent.

There are tables for present value factors just as there are tables for future value factors, and you use them in the same way (if you use them at all). Table 5.3 contains a small set. A much larger set can be found in Table A.2 in the book's appendix.

In Table 5.3, the discount factor we just calculated (.6575) can be found by looking down the column labeled "15%" until you come to the third row.

CALCULATOR HINTS

You solve present value problems on a financial calculator just as you do future value problems. For the example we just examined (the present value of \$1,000 to be received in three years at 15 percent), you would do the following:

	N	I/Y	PMT	PV	FV
Enter	3	15			1,000
Solve for				−657.50	

Notice that the answer has a negative sign; as we discussed earlier, that's because it represents an outflow today in exchange for the \$1,000 inflow later.

EXAMPLE 5.7 Deceptive Advertising?

Businesses sometimes advertise that you should "Come try our product. If you do, we'll give you \$100 just for coming by!" If you read the fine print, what you find out is that they will give you a savings certificate that will pay you \$100 in 25 years or so. If the going interest rate on such certificates is 10 percent per year, how much are they really giving you today?

What you're actually getting is the present value of \$100 to be paid in 25 years. If the discount rate is 10 percent per year, then the discount factor is:

$$1/1.1^{25} = 1/10.8347 = .0923$$

This tells you that a dollar in 25 years is worth a little more than nine cents today, assuming a 10 percent discount rate. Given this, the promotion is actually paying you about $.0923 \times \$100 = \$9.23$. Maybe this is enough to draw customers, but it's not \$100.

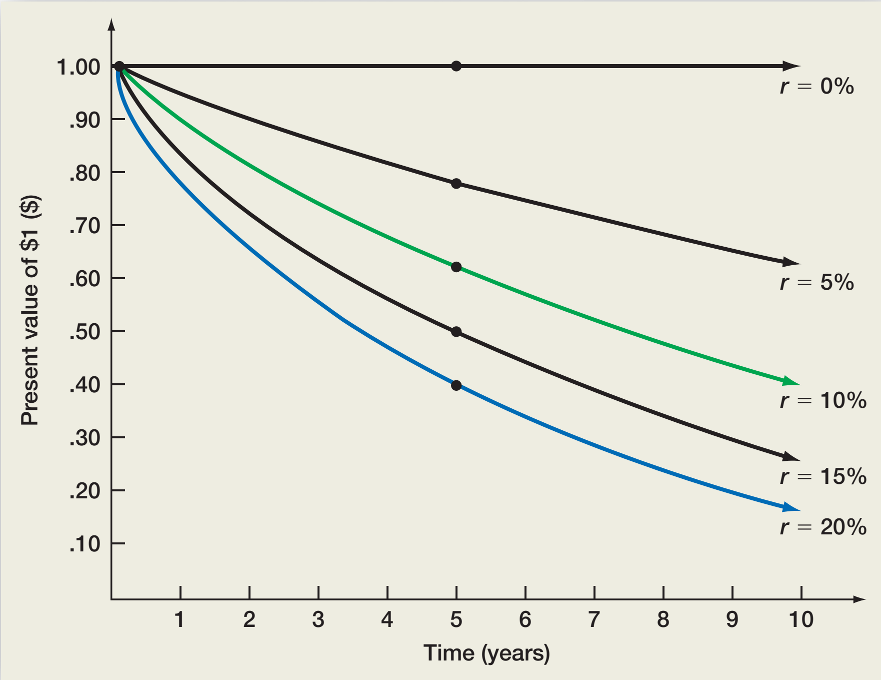


FIGURE 5.3

Present Value of $1 for Different Periods and Rates

As the length of time until payment grows, present values decline. As Example 5.7 illustrates, present values tend to become small as the time horizon grows. If you look out far enough, they will always approach zero. Also, for a given length of time, the higher the discount rate is, the lower is the present value. Put another way, present values and discount rates are inversely related. Increasing the discount rate decreases the PV and vice versa.

The relationship between time, discount rates, and present values is illustrated in Figure 5.3. Notice that by the time we get to 10 years, the present values are all substantially smaller than the future amounts.

Concept Questions

5.2a What do we mean by the present value of an investment?

5.2b The process of discounting a future amount back to the present is the opposite of doing what?

5.2c What do we mean by discounted cash flow, or DCF, valuation?

5.2d In general, what is the present value of $1 to be received in *t* periods, assuming a discount rate of *r* per period?

More about Present and Future Values 5.3

If you look back at the expressions we came up with for present and future values, you will see a simple relationship between the two. We explore this relationship and some related issues in this section.

PRESENT VERSUS FUTURE VALUE

What we called the present value factor is just the reciprocal of (that is, 1 divided by) the future value factor:

$$\text{Future value factor} = (1 + r)^t$$
$$\text{Present value factor} = 1/(1 + r)^t$$

In fact, the easy way to calculate a present value factor on many calculators is to first calculate the future value factor and then press the "1/x" key to flip it over.

If we let FV_t stand for the future value after t periods, then the relationship between future value and present value can be written simply as one of the following:

$$\begin{aligned} PV \times (1 + r)^t &= FV_t \\ PV = FV_t/(1 + r)^t &= FV_t \times [1/(1 + r)^t] \end{aligned} \qquad [5.3]$$

This last result we will call the *basic present value equation*. We will use it throughout the text. A number of variations come up, but this simple equation underlies many of the most important ideas in corporate finance.

EXAMPLE 5.8 Evaluating Investments

To give you an idea of how we will be using present and future values, consider the following simple investment. Your company proposes to buy an asset for $335. This investment is very safe. You would sell off the asset in three years for $400. You know you could invest the $335 elsewhere at 10 percent with very little risk. What do you think of the proposed investment?

This is not a good investment. Why not? Because you can invest the $335 elsewhere at 10 percent. If you do, after three years it will grow to:

$$\begin{aligned} \$335 \times (1 + r)^t &= \$335 \times 1.1^3 \\ &= \$335 \times 1.331 \\ &= \$445.89 \end{aligned}$$

Because the proposed investment pays out only $400, it is not as good as other alternatives we have. Another way of seeing the same thing is to notice that the present value of $400 in three years at 10 percent is:

$$\$400 \times [1/(1 + r)^t] = \$400/1.1^3 = \$400/1.331 = \$300.53$$

This tells us that we have to invest only about $300 to get $400 in three years, not $335. We will return to this type of analysis later on.

DETERMINING THE DISCOUNT RATE

For a downloadable, Windows-based financial calculator, go to www.calculator.org.

We frequently need to determine what discount rate is implicit in an investment. We can do this by looking at the basic present value equation:

$$PV = FV_t/(1 + r)^t$$

There are only four parts to this equation: the present value (PV), the future value (FV_t), the discount rate (r), and the life of the investment (t). Given any three of these, we can always find the fourth.

Finding r for a Single-Period Investment

EXAMPLE 5.9

You are considering a one-year investment. If you put up $1,250, you will get back $1,350. What rate is this investment paying?

First, in this single-period case, the answer is fairly obvious. You are getting a total of $100 in addition to your $1,250. The implicit rate on this investment is thus $100/1,250 = 8 percent.

More formally, from the basic present value equation, the present value (the amount you must put up today) is $1,250. The future value (what the present value grows to) is $1,350. The time involved is one period, so we have:

$$\$1{,}250 = \$1{,}350/(1 + r)^1$$
$$1 + r = \$1{,}350/1{,}250 = 1.08$$
$$r = 8\%$$

In this simple case, of course, there was no need to go through this calculation. But as we describe next, it gets a little harder with more than one period.

To illustrate what happens with multiple periods, let's say we are offered an investment that costs us $100 and will double our money in eight years. To compare this to other investments, we would like to know what discount rate is implicit in these numbers. This discount rate is called the *rate of return,* or sometimes just the *return,* on the investment. In this case, we have a present value of $100, a future value of $200 (double our money), and an eight-year life. To calculate the return, we can write the basic present value equation as:

$$\text{PV} = \text{FV}_t/(1 + r)^t$$
$$\$100 = \$200/(1 + r)^8$$

It could also be written as:

$$(1 + r)^8 = \$200/100 = 2$$

We now need to solve for r. There are three ways we could do it:

1. Use a financial calculator.
2. Solve the equation for $1 + r$ by taking the eighth root of both sides. Because this is the same thing as raising both sides to the power of ⅛ or .125, this is actually easy to do with the "y^x" key on a calculator. Just enter 2, then press "y^x," enter .125, and press the "=" key. The eighth root should be about 1.09, which implies that r is 9 percent.
3. Use a future value table. The future value factor after eight years is equal to 2. If you look across the row corresponding to eight periods in Table A.1, you will see that a future value factor of 2 corresponds to the 9 percent column, again implying that the return here is 9 percent.

Actually, in this particular example, there is a useful "back of the envelope" means of solving for r—the Rule of 72. For reasonable rates of return, the time it takes to double your money is given approximately by $72/r\%$. In our example, this means that $72/r\% =$ eight years, implying that r is 9 percent, as we calculated. This rule is fairly accurate for discount rates in the 5 percent to 20 percent range.

EXAMPLE 5.10 Baseball Collectibles as Investments

In April 2008, the last baseball hit for a home run by Barry Bonds was auctioned off for about $376,000. The price of the ball was considered a bargain, in part because potential buyers were unsure if Bonds would play again. "Experts" on such collectibles often argue that collectibles such as this will double in value over a 10-year period.

So would the ball have been a good investment? By the Rule of 72, you already know the experts were predicting that the ball would double in value in 10 years; so the return predicted would be about 72/10 = 7.2 percent per year, which is only so-so.

At one time at least, a rule of thumb in the rarefied world of fine art collecting was "your money back in 5 years, double your money in 10 years." Given this, let's see how an investment stacked up. In 1998, the Alberto Giacometti bronze statue *L'Homme Qui Marche III* sold for $2,972,500. Five years later, the statue was sold again, walking out the door at a price of $4,039,500. How did the seller do?

Why does the Rule of 72 work? See www.moneychimp.com.

The rule of thumb has us doubling our money in 10 years; so, from the Rule of 72, we have that 7.2 percent per year was the norm. The statue was resold in almost exactly five years. The present value is $2,972,500, and the future value is $4,039,500. We need to solve for the unknown rate, r, as follows:

$$\$2{,}972{,}500 = \$4{,}039{,}500/(1 + r)^5$$
$$(1 + r)^5 = 1.3590$$

Solving for r, we find the seller earned about 6.33 percent per year—less than the 7.2 percent rule of thumb. At least the seller made his money back.

What about other collectibles? To a philatelist (a stamp collector to you and us), one of the most prized stamps is the 1918 24-cent inverted Jenny C3a. The stamp is a collectible because it has a picture of an upside-down biplane. One of these stamps sold at auction for $345,000 in 2010. At what rate did its value grow? Verify for yourself that the answer is about 16.66 percent, assuming a 92-year period.

Other famous collectibles exist because of mistakes. For example, there is the 1944D Lincoln penny, which was struck in zinc rather than copper. In 2010, one of these pennies sold for $60,375. Assuming that 66 years had passed, see if you don't agree that this collectible gained a whopping 26.69 percent per year.

Not all collectibles do as well. Also in 2010, a 1796 $10 gold coin was auctioned for $57,500. While this seems like a huge return to the untrained eye, check that, over the 214-year period, the gain was only about 4.13 percent per year.

Perhaps the most desired coin for numismatists (coin collectors) is the 1933 $20 gold double eagle. Outside of the U.S. Mint and the Smithsonian, only one of these coins is in circulation. In 2002, the coin sold at auction for $7,590,020. See if you agree that this collectible gained about 20.5 percent per year.

A slightly more extreme example involves money bequeathed by Benjamin Franklin, who died on April 17, 1790. In his will, he gave 1,000 pounds sterling to Massachusetts and the city of Boston. He gave a like amount to Pennsylvania and the city of Philadelphia. The money had been paid to Franklin when he held political office, but he believed that politicians should not be paid for their service (it appears that this view is not widely shared by modern politicians).

Franklin originally specified that the money should be paid out 100 years after his death and used to train young people. Later, however, after some legal wrangling, it was agreed that the money would be paid out in 1990, 200 years after Franklin's death. By that time, the Pennsylvania bequest had grown to about \$2 million; the Massachusetts bequest had grown to \$4.5 million. The money was used to fund the Franklin Institutes in Boston and Philadelphia. Assuming that 1,000 pounds sterling was equivalent to \$1,000, what rate of return did the two states earn? (The dollar did not become the official U.S. currency until 1792.)

For Pennsylvania, the future value is \$2 million and the present value is \$1,000. There are 200 years involved, so we need to solve for r in the following:

$$\$1{,}000 = \$2 \text{ million}/(1 + r)^{200}$$
$$(1 + r)^{200} = 2{,}000$$

Solving for r, we see that the Pennsylvania money grew at about 3.87 percent per year. The Massachusetts money did better; verify that the rate of return in this case was 4.3 percent. Small differences in returns can add up!

CALCULATOR HINTS

We can illustrate how to calculate unknown rates using a financial calculator with these numbers. For Pennsylvania, you would do the following:

	N	I/Y	PMT	PV	FV
Enter	200			−1,000	2,000,000
Solve for		3.87			

As in our previous examples, notice the minus sign on the present value, representing Franklin's outlay made many years ago. What do you change to work the problem for Massachusetts?

Saving for College — EXAMPLE 5.11

You estimate that you will need about \$80,000 to send your child to college in eight years. You have about \$35,000 now. If you can earn 20 percent per year, will you make it? At what rate will you just reach your goal?

If you can earn 20 percent, the future value of your \$35,000 in eight years will be:

$$\text{FV} = \$35{,}000 \times 1.20^8 = \$35{,}000 \times 4.2998 = \$150{,}493.59$$

So, you will make it easily. The minimum rate is the unknown r in the following:

$$\text{FV} = \$35{,}000 \times (1 + r)^8 = \$80{,}000$$
$$(1 + r)^8 = \$80{,}000/35{,}000 = 2.2857$$

Therefore, the future value factor is 2.2857. Looking at the row in Table A.1 that corresponds to eight periods, we see that our future value factor is roughly halfway between the ones shown for 10 percent (2.1436) and 12 percent (2.4760), so you will just reach your goal if you earn approximately 11 percent. To get the exact answer, we could use a financial calculator or we could solve for r:

$$(1 + r)^8 = \$80{,}000/35{,}000 = 2.2857$$
$$1 + r = 2.2857^{(1/8)} = 2.2857^{.125} = 1.1089$$
$$r = 10.89\%$$

EXAMPLE 5.12 **Only 18,262.5 Days to Retirement**

You would like to retire in 50 years as a millionaire. If you have \$10,000 today, what rate of return do you need to earn to achieve your goal?

The future value is \$1,000,000. The present value is \$10,000, and there are 50 years until payment. We need to calculate the unknown discount rate in the following:

$$\$10{,}000 = \$1{,}000{,}000/(1 + r)^{50}$$
$$(1 + r)^{50} = 100$$

The future value factor is thus 100. You can verify that the implicit rate is about 9.65 percent.

How much do you need at retirement? Check out the "Money in Your 20s" link at www.about.com.

Not taking the time value of money into account when computing growth rates or rates of return often leads to some misleading numbers in the real world. For example, the most loved (and hated) team in baseball, the New York Yankees, had the highest payroll during the 1988 season, about \$19 million. In 2010, the Yankees again had the highest payroll, a staggering \$206 million: an increase of 984 percent! If history is any guide, we can get a rough idea of the future growth in baseball payrolls. See if you don't agree that this represents an annual increase of 11.4 percent, a substantial growth rate, but much less than the gaudy 984 percent.

How about classic maps? A few years ago, the first map of America, printed in Rome in 1507, was valued at about \$135,000, 69 percent more than the \$80,000 it was worth 10 years earlier. Your return on investment if you were the proud owner of the map over those 10 years? Verify that it's about 5.4 percent per year—far worse than the 69 percent reported increase in price.

Whether with maps or baseball payrolls, it's easy to be misled when returns are quoted without considering the time value of money. However, it's not just the uninitiated who are guilty of this slight form of deception. The title of a feature article in a leading business magazine predicted the Dow Jones Industrial Average would soar to a 70 percent gain over the coming five years. Do you think it meant a 70 percent return per year on your money? Think again!

FINDING THE NUMBER OF PERIODS

Suppose we are interested in purchasing an asset that costs \$50,000. We currently have \$25,000. If we can earn 12 percent on this \$25,000, how long until we have the \$50,000? Finding the answer involves solving for the last variable in the basic present value equation, the number of periods. You already know how to get an approximate answer to this particular problem. Notice that we need to double our money. From the Rule of 72, this will take about $72/12 = 6$ years at 12 percent.

To come up with the exact answer, we can again manipulate the basic present value equation. The present value is \$25,000, and the future value is \$50,000. With a 12 percent discount rate, the basic equation takes one of the following forms:

$$\$25{,}000 = \$50{,}000/1.12^t$$
$$\$50{,}000/25{,}000 = 1.12^t = 2$$

We thus have a future value factor of 2 for a 12 percent rate. We now need to solve for t. If you look down the column in Table A.1 that corresponds to 12 percent, you will see that a future value factor of 1.9738 occurs at six periods. It will thus take about six years, as we calculated. To get the exact answer, we have to explicitly solve for t (or use a financial calculator). If you do this, you will see that the answer is 6.1163 years, so our approximation was quite close in this case.

CALCULATOR HINTS

If you use a financial calculator, here are the relevant entries:

	N	I/Y	PMT	PV	FV
Enter		12		−25,000	50,000
Solve for	6.1163				

Waiting for Godot

EXAMPLE 5.13

You've been saving up to buy the Godot Company. The total cost will be \$10 million. You currently have about \$2.3 million. If you can earn 5 percent on your money, how long will you have to wait? At 16 percent, how long must you wait?

At 5 percent, you'll have to wait a long time. From the basic present value equation:

$$\$2.3 \text{ million} = \$10 \text{ million}/1.05^t$$
$$1.05^t = 4.35$$
$$t = 30 \text{ years}$$

At 16 percent, things are a little better. Verify for yourself that it will take about 10 years.

U.S. EE Savings Bonds are a familiar investment for many. You purchase them for half of their \$100 face value. In other words, you pay \$50 today and get \$100 at some point in the future when the bond "matures." You receive no interest in between, and the interest rate is adjusted every six months, so the length of time until your \$50 grows to \$100 depends on future interest rates. However, at worst, the bonds are guaranteed to be worth \$100 at the end of 17 years, so this is the longest you would ever have to wait. If you do have to wait the full 17 years, what rate do you earn?

Learn more about using Excel for time value and other calculations at www.studyfinance.com.

Because this investment is doubling in value in 17 years, the Rule of 72 tells you the answer right away: $72/17 = 4.24\%$. Remember, this is the minimum guaranteed return, so you might do better. This example finishes our introduction to basic time value concepts. Table 5.4 summarizes present and future value calculations for future reference. As our

TABLE 5.4 Summary of Time Value Calculations

I.	**Symbols:**
	PV = Present value, what future cash flows are worth today
	FV_t = Future value, what cash flows are worth in the future
	r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year
	t = Number of periods—typically, but not always, the number of years
	C = Cash amount
II.	**Future Value of *C* Invested at *r* Percent for *t* Periods:**
	$FV_t = C \times (1 + r)^t$
	The term $(1 + r)^t$ is called the *future value factor*.
III.	**Present Value of *C* to Be Received in *t* Periods at *r* Percent per Period:**
	$PV = C/(1 + r)^t$
	The term $1/(1 + r)^t$ is called the *present value factor*.
IV.	**The Basic Present Value Equation Giving the Relationship between Present and Future Value:**
	$PV = FV_t/(1 + r)^t$

SPREADSHEET STRATEGIES

Using a Spreadsheet for Time Value of Money Calculations

More and more, businesspeople from many different areas (not just finance and accounting) rely on spreadsheets to do all the different types of calculations that come up in the real world. As a result, in this section, we will show you how to use a spreadsheet to handle the various time value of money problems we presented in this chapter. We will use Microsoft Excel™, but the commands are similar for other types of software. We assume you are already familiar with basic spreadsheet operations.

As we have seen, you can solve for any one of the following four potential unknowns: future value, present value, the discount rate, or the number of periods. With a spreadsheet, there is a separate formula for each. In Excel, these are shown in a nearby box.

To Find	Enter This Formula
Future value	= FV (rate,nper,pmt,pv)
Present value	= PV (rate,nper,pmt,fv)
Discount rate	= RATE (nper,pmt,pv,fv)
Number of periods	= NPER (rate,pmt,pv,fv)

In these formulas, pv and fv are present and future value, nper is the number of periods, and rate is the discount, or interest, rate.

Two things are a little tricky here. First, unlike a financial calculator, the spreadsheet requires that the rate be entered as a decimal. Second, as with most financial calculators, you have to put a negative sign on either the present value or the future value to solve for the rate or the number of periods. For the same reason, if you solve for a present value, the answer will have a negative sign unless you input a negative future value. The same is true when you compute a future value.

To illustrate how you might use these formulas, we will go back to an example in the chapter. If you invest $25,000 at 12 percent per year, how long until you have $50,000? You might set up a spreadsheet like this:

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet for time value of money calculations**							
3								
4	If we invest $25,000 at 12 percent, how long until we have $50,000? We need to solve							
5	for the unknown number of periods, so we use the formula NPER(rate,pmt,pv,fv).							
6								
7	Present value (pv):	$25,000						
8	Future value (fv):	$50,000						
9	Rate (rate):	.12						
10								
11	Periods:	**6.1162554**						
12								
13	The formula entered in cell B11 is =NPER(B9,0,-B7,B8); notice that pmt is zero and that pv							
14	has a negative sign on it. Also notice that rate is entered as a decimal, not a percentage.							

nearby *Work the Web* box shows, online calculators are widely available to handle these calculations; however, it is still important to know what is really going on.

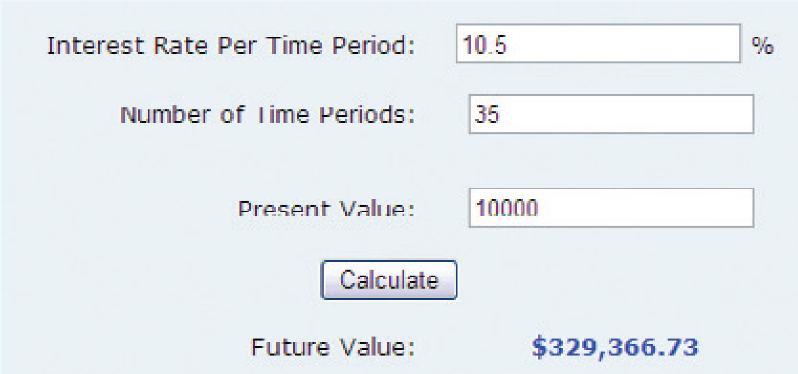
Concept Questions

5.3a What is the basic present value equation?

5.3b What is the Rule of 72?

WORK THE WEB

How important is the time value of money? A recent search on one Web search engine returned over 26.5 million hits! Although you must understand the calculations behind the time value of money, the advent of financial calculators and spreadsheets has eliminated the need for tedious calculations. In fact, many websites offer time value of money calculators. The following is one example from www.investopedia.com. You have $10,000 today and will invest it at 10.5 percent for 35 years. How much will it be worth at that time? With the Investopedia calculator, you simply enter the values and hit "Calculate". The results look like this:



Who said time value of money calculations are hard?

Questions

1. *Use the present value calculator on this website to answer the following: Suppose you want to have $140,000 in 25 years. If you can earn a 10 percent return, how much do you have to invest today?*
2. *Use the future value calculator on this website to answer the following question: Suppose you have $8,000 today that you plan to save for your retirement in 40 years. If you earn a return of 10.8 percent per year, how much will this account be worth when you are ready to retire?*

Summary and Conclusions 5.4

This chapter has introduced you to the basic principles of present value and discounted cash flow valuation. In it, we explained a number of things about the time value of money, including these:

1. For a given rate of return, we can determine the value at some point in the future of an investment made today by calculating the future value of that investment.
2. We can determine the current worth of a future cash flow or series of cash flows for a given rate of return by calculating the present value of the cash flow(s) involved.

3. The relationship between present value (PV) and future value (FV) for a given rate r and time t is given by the basic present value equation:

$$PV = FV_t/(1 + r)^t$$

As we have shown, it is possible to find any one of the four components (PV, FV_t, r, or t) given the other three.

The principles developed in this chapter will figure prominently in the chapters to come. The reason for this is that most investments, whether they involve real assets or financial assets, can be analyzed using the discounted cash flow (DCF) approach. As a result, the DCF approach is broadly applicable and widely used in practice. Before going on, therefore, you might want to do some of the problems that follow.

CONNECT TO FINANCE

Can you answer the following questions?

Section 5.1 You deposited $2,000 in a bank account that pays 5 percent simple interest. How much will you have in this account after three years? (LO 5-1)

Section 5.2 What is the present value of $11,500 discounted at 9 percent for 11 years? (LO 5-2)

Section 5.3 Charlie invested $6,200 in a stock last year. Currently, this investment is worth $6,788.38. What is the rate of return on this investment? (LO 5-3)

Log on to find out!

CHAPTER REVIEW AND SELF-TEST PROBLEMS

5.1 Calculating Future Values Assume you deposit $10,000 today in an account that pays 6 percent interest. How much will you have in five years?

5.2 Calculating Present Values Suppose you have just celebrated your 19th birthday. A rich uncle has set up a trust fund for you that will pay you $150,000 when you turn 30. If the relevant discount rate is 9 percent, how much is this fund worth today?

5.3 Calculating Rates of Return You've been offered an investment that will double your money in 10 years. What rate of return are you being offered? Check your answer using the Rule of 72.

5.4 Calculating the Number of Periods You've been offered an investment that will pay you 9 percent per year. If you invest \$15,000, how long until you have \$30,000? How long until you have \$45,000?

ANSWERS TO CHAPTER REVIEW AND SELF-TEST PROBLEMS

5.1 We need to calculate the future value of \$10,000 at 6 percent for five years. The future value factor is:

$$1.06^5 = 1.3382$$

The future value is thus \$10,000 × 1.3382 = \$13,382.26.

5.2 We need the present value of \$150,000 to be paid in 11 years at 9 percent. The discount factor is:

$$1/1.09^{11} = 1/2.5804 = .3875$$

The present value is thus about \$58,130.

5.3 Suppose you invest \$1,000. You will have \$2,000 in 10 years with this investment. So, \$1,000 is the amount you have today, or the present value, and \$2,000 is the amount you will have in 10 years, or the future value. From the basic present value equation, we have:

$$\$2{,}000 = \$1{,}000 \times (1 + r)^{10}$$
$$2 = (1 + r)^{10}$$

From here, we need to solve for r, the unknown rate. As shown in the chapter, there are several different ways to do this. We will take the 10th root of 2 (by raising 2 to the power of 1/10):

$$2^{(1/10)} = 1 + r$$
$$1.0718 = 1 + r$$
$$r = 7.18\%$$

Using the Rule of 72, we have $72/t = r\%$, or $72/10 = 7.2\%$; so, our answer looks good (remember that the Rule of 72 is only an approximation).

5.4 The basic equation is this:

$$\$30{,}000 = \$15{,}000 \times (1 + .09)^t$$
$$2 = (1 + .09)^t$$

If we solve for t, we find that $t = 8.04$ years. Using the Rule of 72, we get $72/9 = 8$ years, so once again our answer looks good. To get \$45,000, verify for yourself that you will have to wait 12.75 years.

CONCEPTS REVIEW AND CRITICAL THINKING QUESTIONS

1. **Present Value [LO2]** The basic present value equation has four parts. What are they?
2. **Compounding [LO1, 2]** What is compounding? What is discounting?
3. **Compounding and Period [LO1]** As you increase the length of time involved, what happens to future values? What happens to present values?

4. **Compounding and Interest Rates [LO1]** What happens to a future value if you increase the rate *r*? What happens to a present value?
5. **Ethical Considerations [LO2]** Take a look back at Example 5.7. Is it deceptive advertising? Is it unethical to advertise a future value like this without a disclaimer?

To answer the next five questions, refer to the TMCC security we discussed to open the chapter.

6. **Time Value of Money [LO2]** Why would TMCC be willing to accept such a small amount today ($24,099) in exchange for a promise to repay about four times that amount ($100,000) in the future?
7. **Call Provisions [LO2]** TMCC has the right to buy back the securities on the anniversary date at a price established when the securities were issued (this feature is a term of this particular deal). What impact does this feature have on the desirability of this security as an investment?
8. **Time Value of Money [LO2]** Would you be willing to pay $24,099 today in exchange for $100,000 in 30 years? What would be the key considerations in answering yes or no? Would your answer depend on who is making the promise to repay?
9. **Investment Comparison [LO2]** Suppose that when TMCC offered the security for $24,099, the U.S. Treasury had offered an essentially identical security. Do you think it would have had a higher or lower price? Why?
10. **Length of Investment [LO2]** The TMCC security is bought and sold on the New York Stock Exchange. If you looked at the price today, do you think the price would exceed the $24,099 original price? Why? If you looked in the year 2019, do you think the price would be higher or lower than today's price? Why?

QUESTIONS AND PROBLEMS

BASIC
(Questions 1–15)

1. **Simple Interest versus Compound Interest [LO1]** First City Bank pays 7 percent simple interest on its savings account balances, whereas Second City Bank pays 7 percent interest compounded annually. If you made a $6,000 deposit in each bank, how much more money would you earn from your Second City Bank account at the end of nine years?
2. **Calculating Future Values [LO1]** For each of the following, compute the future value:

Present Value	Years	Interest Rate	Future Value
$ 2,250	11	13%	
8,752	7	9	
76,355	14	12	
183,796	8	6	

3. **Calculating Present Values [LO2]** For each of the following, compute the present value:

Present Value	Years	Interest Rate	Future Value
	13	7%	$ 15,451
	4	13	51,557
	29	14	886,073
	40	9	550,164

4. **Calculating Interest Rates [LO3]** Solve for the unknown interest rate in each of the following:

Present Value	Years	Interest Rate	Future Value
$ 240	4		$ 297
360	18		1,080
39,000	19		185,382
38,261	25		531,618

5. **Calculating the Number of Periods [LO4]** Solve for the unknown number of years in each of the following:

Present Value	Years	Interest Rate	Future Value
$ 560		9%	$ 1,389
810		10	1,821
18,400		17	289,715
21,500		15	430,258

6. **Calculating Interest Rates [LO3]** Assume the total cost of a college education will be $300,000 when your child enters college in 18 years. You presently have $65,000 to invest. What annual rate of interest must you earn on your investment to cover the cost of your child's college education?
7. **Calculating the Number of Periods [LO4]** At 6.5 percent interest, how long does it take to double your money? To quadruple it?
8. **Calculating Interest Rates [LO3]** In January 2010, the average house price in the United States was $283,400. In January 2000, the average price was $200,300. What was the annual increase in selling price?
9. **Calculating the Number of Periods [LO4]** You're trying to save to buy a new $190,000 Ferrari. You have $40,000 today that can be invested at your bank. The bank pays 4.8 percent annual interest on its accounts. How long will it be before you have enough to buy the car?
10. **Calculating Present Values [LO2]** Imprudential, Inc., has an unfunded pension liability of $575 million that must be paid in 20 years. To assess the value of the firm's stock, financial analysts want to discount this liability back to the present. If the relevant discount rate is 6.8 percent, what is the present value of this liability?
11. **Calculating Present Values [LO2]** You have just received notification that you have won the $1 million first prize in the Centennial Lottery. However, the prize will be awarded on your 100th birthday (assuming you're around to collect), 80 years from now. What is the present value of your windfall if the appropriate discount rate is 9 percent?
12. **Calculating Future Values [LO1]** Your coin collection contains fifty 1952 silver dollars. If your grandparents purchased them for their face value when they were new, how much will your collection be worth when you retire in 2060, assuming they appreciate at a 4.1 percent annual rate?
13. **Calculating Interest Rates and Future Values [LO1, 3]** In 1895, the first U.S. Open Golf Championship was held. The winner's prize money was $150. In 2010, the winner's check was $1,350,000. What was the percentage increase per year in the winner's check over this period? If the winner's prize increases at the same rate, what will it be in 2040?

14. **Calculating Interest Rates [LO3]** In 2010, a gold Morgan dollar minted in 1895 sold for $125,000. For this to have been true, what rate of return did this coin return for the lucky numismatist?

15. **Calculating Rates of Return [LO3]** Although appealing to more refined tastes, art as a collectible has not always performed so profitably. During 2003, Sotheby's sold the Edgar Degas bronze sculpture *Petite Danseuse de Quatorze Ans* at auction for a price of $10,311,500. Unfortunately for the previous owner, he had purchased it in 1999 at a price of $12,377,500. What was his annual rate of return on this sculpture?

INTERMEDIATE
(Questions 16–20)

16. **Calculating Rates of Return [LO3]** Referring to the TMCC security we discussed at the very beginning of the chapter:
    a. Based on the $24,099 price, what rate was TMCC paying to borrow money?
    b. Suppose that, on March 28, 2019, this security's price is $42,380. If an investor had purchased it for $24,099 at the offering and sold it on this day, what annual rate of return would she have earned?
    c. If an investor had purchased the security at market on March 28, 2019, and held it until it matured, what annual rate of return would she have earned?

17. **Calculating Present Values [LO2]** Suppose you are still committed to owning a $190,000 Ferrari (see Problem 9). If you believe your mutual fund can achieve a 12 percent annual rate of return and you want to buy the car in 9 years on the day you turn 30, how much must you invest today?

18. **Calculating Future Values [LO1]** You have just made your first $5,000 contribution to your retirement account. Assuming you earn an 11 percent rate of return and make no additional contributions, what will your account be worth when you retire in 45 years? What if you wait 10 years before contributing? (Does this suggest an investment strategy?)

19. **Calculating Future Values [LO1]** You are scheduled to receive $15,000 in two years. When you receive it, you will invest it for six more years at 7.1 percent per year. How much will you have in eight years?

20. **Calculating the Number of Periods [LO4]** You expect to receive $15,000 at graduation in two years. You plan on investing it at 11 percent until you have $85,000. How long will you wait from now?